# A guide to the experimental data for $\pi^0$, $\eta$ and $\eta'$ transition form factors and decay distributions

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**Abstract**

Overview of experimental data on meson transition form factors.

# 1 Introduction

The scope of the guide is to collect references to the experiments relevant to transition form factor of the lightest pseudoscalar mesons $\pi^0$, $\eta$ and $\eta'$. This covers in first place anomalous processes involving odd number of pseudoscalar fields and a vector (photon or a vector meson) particle. The focus of the review is on experimental distributions of the processes and thus the informations which are not normally contained in the particle properties reviews of the Particle Data Group. This project is follow up of MesonNet workshop on Transition Form Factors [1].

A Compilation of data on hadronic total cross-sections in $e^+e^-$ interactions is available [2] on Hepdb `http://durpdg.dur.ac.uk/hepdata/online/rsig/index.html`

In addition to the anomalous processes we include also informations on processes with even number of the pseudoscalar fields such as $\eta \to \pi^+\pi^-\pi^0$ since representation of the dynamics of such processes should be done using Dalitz plot analysis. This is motivated by theory request to have directly access to Dalitz plot data instead of using parameters introduced by experiments.

Comments concerning inclusion of radiative corrections, using of Dalitz plots are included.

Two gamma processes reviews: [3, 4].

In the outlook we discuss further experiments possible with existing facilities.

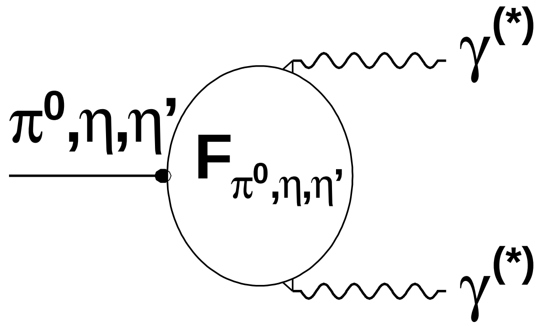


Figure 1: General $P\gamma^*\gamma^*$ vertex described by a transition form factor.

## 2 Definitions

A transition form factor $\mathcal{F}_P(q_1^2, q_2^2)$ is a scalar function of the four-momentum transfer squared of the virtual photons ($q_{1,2}^2$) describing the vertex in Fig. 1 and defined as

$$\mathcal{A}(P \to \gamma^*\gamma^{(*)}) = q_1^\mu \epsilon_1^\nu q_2^\alpha \epsilon_2^\beta \epsilon_{\mu\nu\alpha\beta} \mathcal{F}_P(q_1^2, q_2^2) \tag{1}$$

and

$$\frac{m_P^3}{64\pi} |\mathcal{F}_P(q_1^2 = 0, q_2^2 = 0)|^2 = \Gamma(P \to \gamma\gamma).$$

Brodsky-Lepage definition for $\gamma^* P \to \gamma$ case [?]:

$$\mathcal{A}_\mu = -ie^2 \mathcal{F}_P(Q^2) \epsilon_{\mu\nu\alpha\beta}$$

[33, 20]

$$\frac{m_P^3}{64\pi} |\mathcal{F}_P(q_1^2 = 0, q_2^2 = 0)|^2 = \frac{\Gamma(P \to \gamma\gamma)}{(4\pi\alpha)^2}.$$

One often uses a normalized transition form factor:

$$F_P(q_1^2, q_2^2) = \frac{\mathcal{F}_P(q_1^2, q_2^2)}{\mathcal{F}_P(q_1^2 = 0, q_2^2 = 0)} \ . \tag{2}$$

For example, for the neutral pion physical processes include

- $\pi^0 \to 2\gamma$: This is well described by the chiral anomaly encoded in the Wess–Zumino–Witten action, see, e.g., [?, ?].
- $\pi^0 \to \gamma e^+ e^-$ and $\pi^0 \to e^+ e^- e^+ e^-$:
- $e^+ e^- \to \pi^0 e^+ e^-$: Here two virtual photons can fuse "in flight" to form the pion.[1]
- Another process involving the pion transition form factor is a very rare direct dilepton decay $\pi^0 \to e^+ e^-$. Two photons emitted in this process convert to a dilepton by lepton exchange, see Fig. 34c in [?]. The pion transition form factor enters the corresponding "QED loop"; see, e.g., [?].

[1] In principle, this process interferes at the amplitude level with $e^+e^-$ annihilation and successive emission of a pion and virtual photon.

| Mode | $\mathcal{B}^{\rm exp}$ | Ref. | $\mathcal{B}^{\rm th}$ | Ref. |
|---|---|---|---|---|
| $\pi^0 \to \gamma\gamma$ | $(98.823 \pm 0.034)\%$ | [5] | – | – |
| $\pi^0 \to e^+e^-\gamma$ | $(1.174 \pm 0.035)\%$ | [5] | $(1.182 \pm 0.003)\%$ | [?] |
| $\pi^0 \to e^+e^-e^+e^-$ | $(3.34 \pm 0.16) \cdot 10^{-5}$ | [5] | – | – |
| $\pi^0 \to e^+e^-$ | $(7.48 \pm 0.29 \pm 0.25) \cdot 10^{-8}$ [2] | [6] | $(6.33 \pm 0.19) \cdot 10^{-8}$ | [7] |
| | | | $(8.3 \pm 0.4) \cdot 10^{-8}$ | [8] |
| | | | $(6.2 \pm 0.1) \cdot 10^{-8}$ | [?] |

Table 1: Branching fractions of $\pi^0$ leptonic decays.

# 3 BR

In Table 1 we list the information on the branching fractions of $\pi^0$ decays together with the corresponding theoretical predictions. The branching ratios follow largely the naive scaling as $1 : \alpha_{\rm QED} : \alpha_{\rm QED}^2 : (\alpha_{\rm QED}/(4\pi)^2)^2$, where the factor $(4\pi)^2$ appears, since in the Standard Model the leading contribution is at one loop. Note, between the most accurate calculation for $\pi^0 \to e^+e^-$ and the corresponding experimental value there is a more than $3\sigma$ discrepancy.

In Tables 2 and 3 we show the branching fractions of the $\eta$ and $\eta'$ purely leptonic decays.

| Mode | $\mathcal{B}^{\rm exp}$ | Ref. | $\mathcal{B}^{\rm th}$ | Ref. |
|---|---|---|---|---|
| $\eta \to \gamma\gamma$ | $(39.31 \pm 0.20)\%$ | [5] | – | – |
| $\eta \to e^+e^-$ | $< 2.7 \cdot 10^{-5}$ | [9] | $(5.8 \pm 0.2) \cdot 10^{-9}$ | [8] |
| $\eta \to \mu^+\mu^-$ | $(5.8 \pm 0.8) \cdot 10^{-6}$ | [5] | – | – |
| $\eta \to e^+e^-\gamma$ | $(7.0 \pm 0.7) \cdot 10^{-3}$ | [5] | – | – |
| $\eta \to \mu^+\mu^-\gamma$ | $(3.1 \pm 0.4) \cdot 10^{-4}$ | [10] | – | – |
| $\eta \to e^+e^-e^+e^-$ | $(2.4 \pm 0.2_{stat+bckg} \pm 0.1_{syst}) \cdot 10^{-5}$ | [11] | – | – |
| $\eta \to e^+e^-\mu^+\mu^-$ | $< 1.6 \cdot 10^{-4}$ | [9] | – | – |
| $\eta \to \mu^+\mu^-\mu^+\mu^-$ | $< 3.6 \cdot 10^{-4}$ | [9][3] | – | – |

Table 2: Branching fractions of $\eta$ leptonic decays

This involves processes like

- $\omega \to \pi^0 l^+l^-$ where $l = e, \mu$: For an illustration see Fig. 2. This process shows a dramatic deviation from the vector meson dominance picture [15, 16], see Fig. 6.

- $\phi$ instead of $\omega$ in the previous processes and/or $\eta$ instead of $\pi^0$ (in part measured): It would be important to clarify whether also in these processes the drastic deviation from vector meson dominance seen in $\omega \to \pi^0\mu^+\mu^-$ shows up. Of particular importance is $\phi \to \pi^0 l^+l^-$ where the peak mass of the $\rho$ meson is in the kinematically allowed region.

[2]The value given in Ref [5], $(6.46 \pm 0.33) \cdot 10^{-8}$, is not corrected for final state radiation.

[3]Since the authors do not distinguish between the $\mu^+\mu^-\mu^+\mu^-$ and $\mu^+\mu^-\pi^+\pi^-$ final states, the upper limit is for a sum of the branching fractions for the two modes.

| Mode | $\mathcal{B}^{\text{exp}}$ | Ref. | $\mathcal{B}^{\text{th}}$ | Ref. |
|---|---|---|---|---|
| $\eta' \to \gamma\gamma$ | $(2.18 \pm 0.08)\%$ | [5] | – | – |
| $\eta' \to e^+e^-$ | $< 2.1 \cdot 10^{-7}$ | [12] | $(1.5 \pm 0.1) \cdot 10^{-10}$ | [8] |
| $\eta' \to \mu^+\mu^-$ | – | – | $(2.1 \pm 0.3) \cdot 10^{-7}$ | [8] |
| $\eta' \to e^+e^-\gamma$ | $< 9 \cdot 10^{-4}$ | [13] | – | – |
| $\eta' \to \mu^+\mu^-\gamma$ | $(1.07 \pm 0.26) \cdot 10^{-4}$ | [14] | – | – |

Table 3: Branching fractions of $\eta'$ leptonic decays.

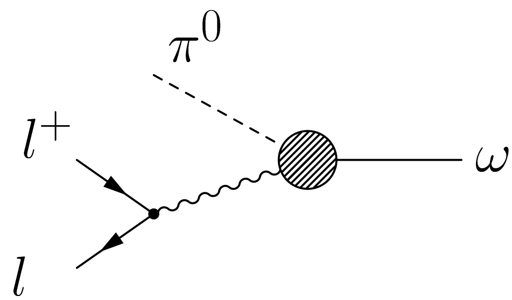


Figure 2: Transition form factor of omega to pion.

- $\eta' \to \omega\gamma$ (measured) and $\eta' \to \omega e^+e^-$ (not measured).

### 3.1 Transition form factors

For asymptotically large virtualities there are QCD constraints on the pion transition form factor, see, e.g. [?, ?, ?]. This might be at odds with recent experimental results from BaBar [20]; see, however, also the recent Belle results [21].

Single o -shell form factors can be studied in:

1. pseudoscalar meson decays $P \to l^+l^-\gamma$ $(4m_l^2 < q^2 < m_P^2)$
2. $e^+e^- \to P\gamma$ $(q^2 > m_P^2)$

$$\sigma(e^+e^- \to P\gamma) = 4\pi\alpha_{\text{QED}}\Gamma_{\gamma\gamma}\left(\frac{s - m_P^2}{sm_P}\right)^3 |F_P(q^2 = s, 0)|^2 \tag{3}$$

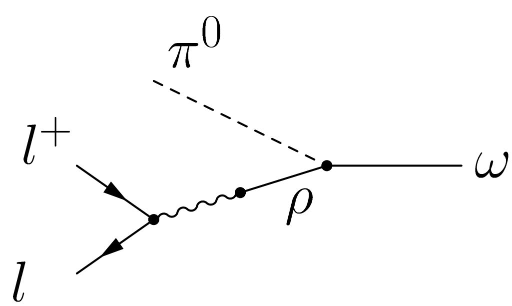


Figure 3: Tree-level contribution to the omega transition form factor.

| Mode | $\mathcal{B}^{\text{exp}}$ | Ref. | $\mathcal{B}^{\text{th}}$ | Ref. |
|---|---|---|---|---|
| $\omega \to \pi^0 e^+ e^-$ | $(7.7 \pm 0.6) \cdot 10^{-4}$ | [5] | – | – |
| $\omega \to \pi^0 \mu^+ \mu^-$ | $(1.73 \pm 0.25 \pm 0.14) \cdot 10^{-4}$ | [15] | – | – |
| | $(0.96 \pm 0.23) \cdot 10^{-4}$ | [17] | – | – |
| $\omega \to \eta e^+ e^-$ | $< 1.1 \cdot 10^{-5}$ | [18] | – | – |

Table 4: Branching fractions of $\omega$ conversion decays.

| Mode | $\mathcal{B}^{\text{exp}}$ | Ref. | $\mathcal{B}^{\text{th}}$ | Ref. |
|---|---|---|---|---|
| $\phi \to \pi^0 e^+ e^-$ | $(1.12 \pm 0.28) \cdot 10^{-5}$ | [5] | – | – |
| $\phi \to \eta e^+ e^-$ | $(1.15 \pm 0.10) \cdot 10^{-4}$ | [5] | – | – |
| $\phi \to \eta \mu^+ \mu^-$ | $< 9.4 \cdot 10^{-6}$ | [19] | – | – |

Table 5: Branching fractions of $\phi$ conversion decays

3. two-photon production: e.g.,: in $e^- e^\pm$ interactions and Primako process. ($q^2 < 0$)

$$\sigma_{\gamma^*\gamma^* \to P} = \frac{16\pi^2}{m_P^3} \Gamma_{\gamma\gamma} |F(q_1^2, q_2^2)|^2 \sqrt{(q_1 \cdot q_2)^2 - q_1^2 q_2^2}\, \delta\left((q_1 + q_2)^2 - m_P^2\right)$$

A transition of the omega meson to pion is given by the two-dimensional $\pi^0$ form factor as: $|F_{\pi^0}(q_1^2, m_\omega^2)|^2 / |F_{\pi^0}(0, m_\omega^2)|^2$. It was measured in $\omega \to \pi^0 \ell^+ \ell^-$ and in $e^+ e^- \to \omega \pi^0-$ reaction. The results are shown in Fig. 6 together with naive VMD predictions. Note that the region around $q_1^2 \approx m_\omega^2$ is just included for completeness. There, the equations (**??**), (**??**) do not provide a good approximation.

## 3.2 Dalitz decays

The $q_{1,2}^2$ for the conversion (Dalitz) decays (Fig. 7) is equal to the invariant mass squared of the lepton-antilepton pair and $m_P^2 \geq q_{1,2}^2 \geq 4m_\ell^2$ (time-like virtual photons). The amplitude of the single conversion decay of a pseudoscalar meson $P$ is given by:

$$\mathcal{A}(P \to \ell^+ \ell^- \gamma) = ie\mathcal{F}_P(q_1^2, 0) \epsilon_{\mu\nu\sigma\tau} \varepsilon_2^\mu q_2^\nu \varepsilon_1^\sigma \, \frac{1}{q_1^2} \, [\bar{u} \gamma^\tau u] \tag{4}$$

where $1/q_1^2$ is the photon propagator and the last term is the leptonic current.

Experimentally the form factor could be extracted from the $q^2$ distribution given by:

$$\frac{d\Gamma(P \to \ell^+ \ell^- \gamma)}{dq^2 \Gamma_{\gamma\gamma}} = \frac{2\alpha}{3\pi} \frac{1}{q^2} \sqrt{1 - \frac{4m_\ell^2}{q^2}} \left(1 + \frac{2m_\ell^2}{q^2}\right) \left(1 - \frac{q^2}{M_P^2}\right)^3 |F_P(q^2, 0)|^2. \tag{5}$$

The distributions for $\eta$ Dalitz decays are presented in Fig. 8. The distributions for the $e^+ e^- \gamma$ final states are peaked at $4m_\ell^2$ due to the $1/q^2$ QED term. The form factor can be obtained by dividing out the QED dependence. To extract form factor slope a dependence

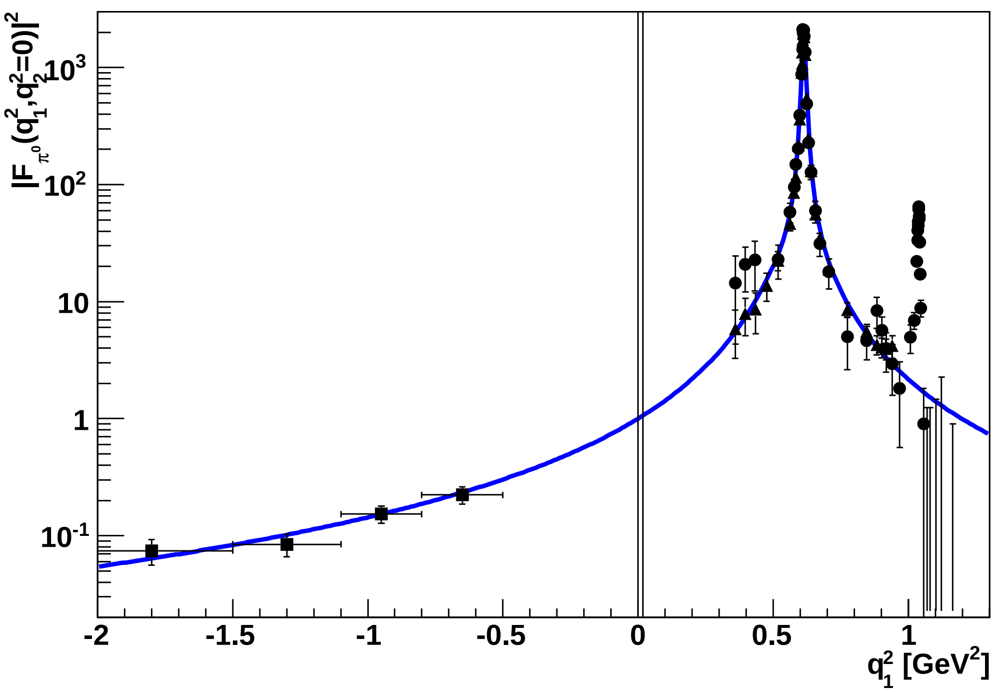


Figure 4: Single o  -shell $\pi^0$ meson transition form factor in the low $|q^2|$ region from SND [22] and CMD-2 [23] data on the reaction $e^+e^- \to \pi^0\gamma$ and CELLO data on the reaction $e^+e^- \to e^+e^-\gamma^*\gamma^* \to e^+e^-\pi^0$ [24]

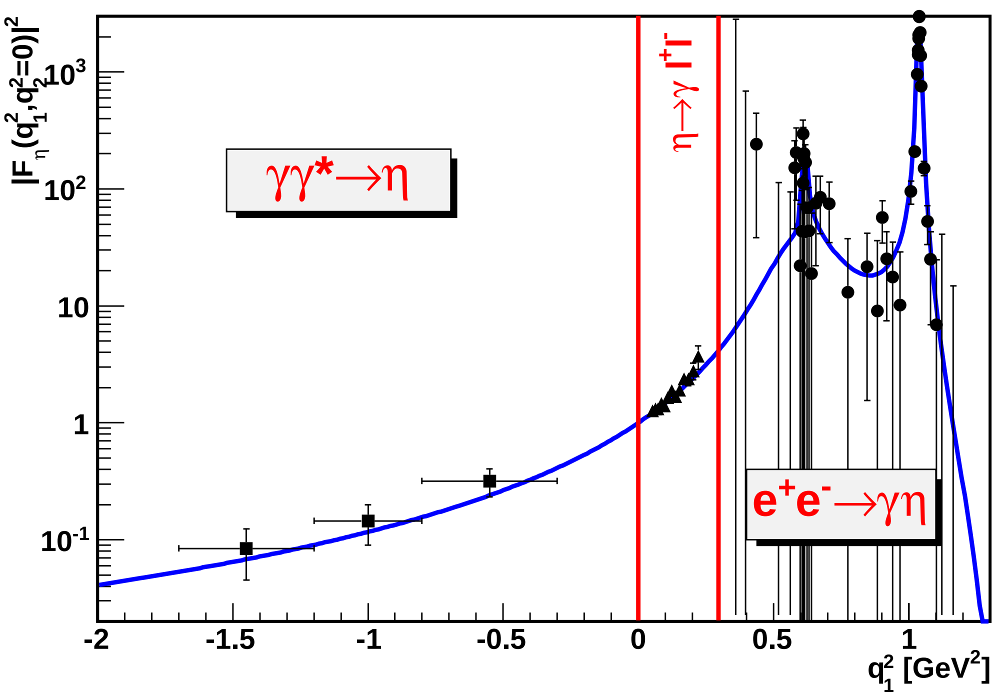


Figure 5: Single o -shell $\eta$ meson transition form factor from NA60 data on $\eta \to \gamma\mu^+\mu^-$ decay [15]; from SND [22] and CMD-2 [23] data on the reaction $e^+e^- \to \eta\gamma$ reaction, and CELLO data on the reaction $e^+e^- \to e^+e^-\gamma^*\gamma^* \to e^+e^-\eta$ [24].

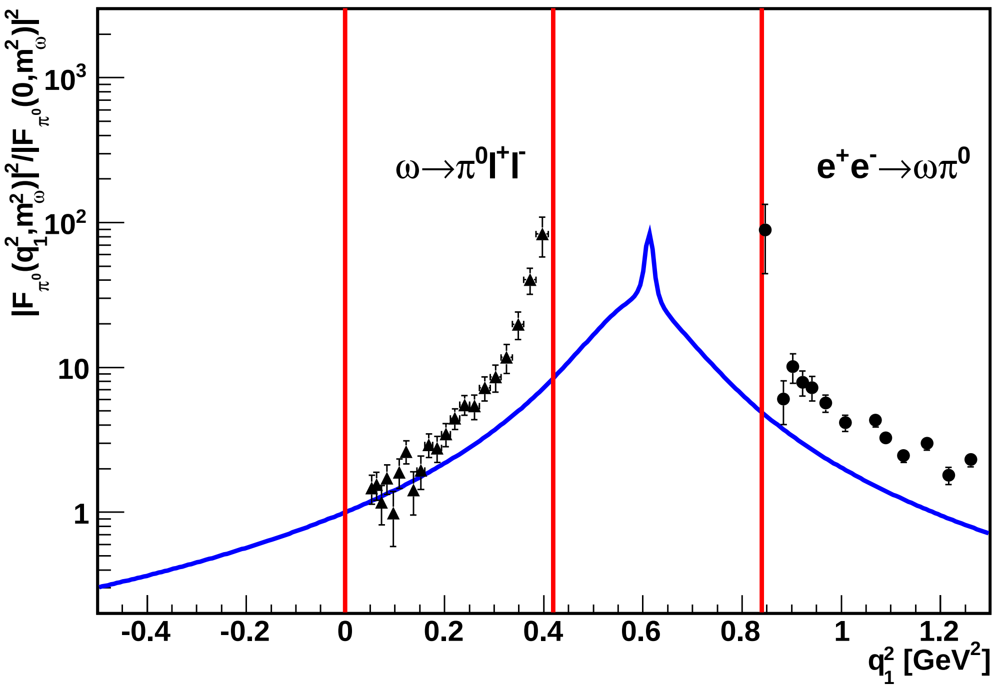


Figure 6: Data on the $\omega$ *transition form factor* $|F_{\pi^0}(q_1^2, m_\omega^2)|^2/|F_{\pi^0}(0, m_\omega^2)|^2$ from NA60 data on $\omega \to \pi^0\mu^+\mu^-$ decay [15] and from SND [25], CMD-2 [26], and KLOE [27] experiments on the $e^+e^- \to \omega\pi^0$ reaction.

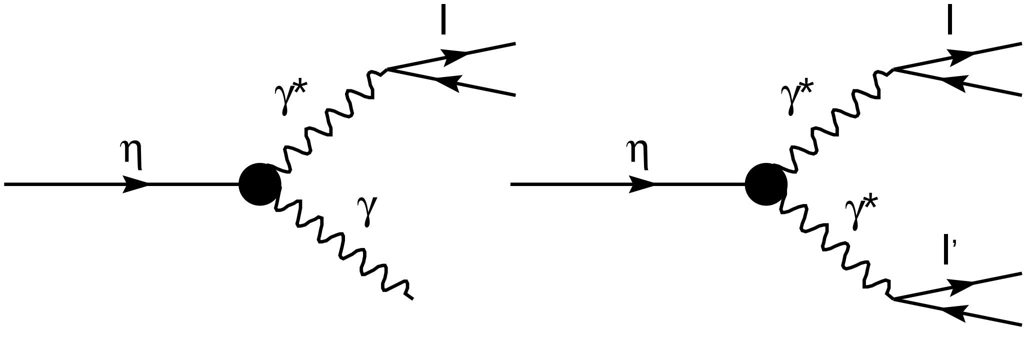


Figure 7: Single and double Dalitz decays

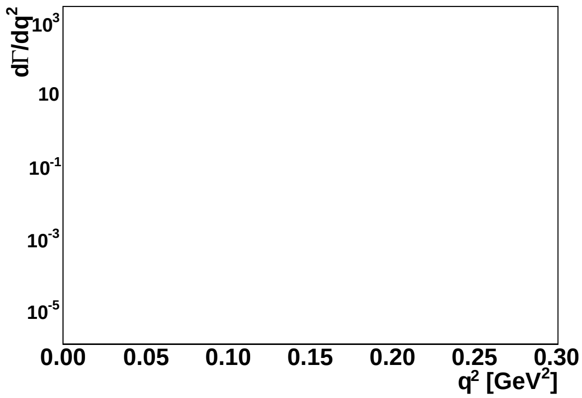


Figure 8: $d\Gamma/dq^2$ distributions for single Dalitz decays of $\eta$ meson: (left) solid line – $\eta \to e^+e^-\gamma$ with $F_\eta(q^2, 0) = 1$; dotted line – $\eta \to \mu^+\mu^-\gamma$ with $F_\eta(q^2) = 1$; dashed line – $\eta \to e^+e^-\gamma$ with the VMD form factor.

| | $b_{\pi^0}(q^2)$ [GeV$^{-2}$] | |
|---|---|---|
| $b_{\pi^0}(q^2 = 0)$ | $1.79 \pm 0.14$ | CELLO [24] |
| $b_{\pi^0}(q^2 = 0.613\text{GeV}^2)$ | $2.36 \pm 0.21$ | Lepton-G [17] |
| $b_{\pi^0}(q^2 = 0.613\text{GeV}^2)$ | $2.24 \pm 0.06 \pm 0.02$ | NA60 [15] |
| $b_{\pi^0}(q^2 = 0.613\text{GeV}^2)$ | $2.241 \pm 0.025 \pm 0.028$ | NA60 [16] |
| | $b_\eta(q^2)$ [GeV$^{-2}$] | |
| $b_\eta(q^2 = 0)$ | $1.9 \pm 0.4$ | Lepton-G [10] |
| $b_\eta(q^2 = 0)$ | $1.42 \pm 0.21$ | CELLO [24] |
| $b_\eta(q^2 = 0\text{GeV}^2)$ | $1.6 \pm 2$ | SND [22] |
| $b_\eta(q^2 = 0)$ | $1.95 \pm 0.17 \pm 0.05$ | NA60 [15] |
| $b_\eta(q^2 = 0)$ | $1.92 \pm 0.35 \pm 0.13$ | CB/TAPS [28] |
| $b_\eta(q^2 = 1.04\text{GeV}^2)$ | $3.8 \pm 1.8$ | SND [22] |

Table 6: Summary of the available experimental data on $b_\eta(q^2)$ and $b_{\pi^0}(q^2)$.

on the $q^2$ variable is fitted with a single-pole formula, which at low energies $q^2 \ll \Lambda^2$ can be approximated successively as follows:

$$F(q^2, 0) = \frac{\Lambda^2}{\Lambda^2 - q^2 - i\Gamma\Lambda} \approx \frac{\Lambda^2}{\Lambda^2 - q^2} \approx 1 + \frac{q^2}{\Lambda^2}. \tag{6}$$

The form factor slope $b_P(0)$ is therefore related to $\Lambda$:

$$b_P(0) \equiv \left.\frac{d \ln F(q^2, 0)}{dq^2}\right|_{q^2=0} = \frac{1}{\Lambda^2}. \tag{7}$$

Available data on $|F_{\pi^0}|$ and $|F_\eta|$ for low $|q^2|$ values are presented in Figs 4, 5, and 6.

# 4 Radiative and hadronic anomalous processes

Finally, there is one more twist: The $\rho$ meson is rather broad and couples to two pions. Therefore processes, which involve two pions instead of a dilepton, are also intimately connected to the previous reactions. This involves, e.g., $\eta \to \pi^+\pi^-\gamma^{(*)}$. There are indications for deviations from vector-meson dominance [29, ?]. (Of course, this logic can be extended to reactions with three pions instead of an $\omega$ or $\phi$. But then there are a lot of di erent channels which interfere with each other.) For a collection of data, a list of possible reactions might be useful. This is provided in the appendix. **DATA on eta,eta' to pi+pi-g![29, 30] prospects eta' BESIII, CLAS**

# A List of processes

The focus is on reactions, which are possible to study in practice. For example, scattering reactions with initial-state neutral pions (like $\pi^0\gamma \to$something) are not considered because there are no $\pi^0$ beams available. There is a charged-pion beam, though, (COMPASS at CERN), and "collisions" with photons can be achieved using the Primako e ect. Likewise, reactions with initial-state muons are not listed, though this is one of the future dreams of high-energy physicists. Reactions, which involve a broad $\rho$ meson, are listed in the following as $\pi^+\pi^-$. In principle, reactions like $e^+e^- \to \pi^+\pi^-$ and $e^+e^- \to \pi^0\omega$ are related to $\tau^- \to \nu_\tau\pi^-\pi^0$ and $\tau^- \to \nu_\tau\pi^-\omega$ through conservation of vector current and isospin symmetry [?]. Decays of the $\tau$ are not listed below.

1. Processes with one external (pseudoscalar) hadron: $\pi^0$

   (a) $\pi^0 \to 2\gamma$: BR,$\Gamma$

   (b) $\pi^0 \to \gamma\, e^+e^-$: BR,$|F(q^2,0)|^2$

   - SINDRUM (1992) [31]
   - Prospects: KTeV, WASA-at-COSY, NA48

   (c) $\pi^0 \to e^+e^-\, e^+e^-$: BR,$|F(q^2,0)|^2$

   (d) $\pi^0 \to e^+e^-$

   (e) $e^+e^- \to \pi^0\,\gamma$:

   - SND (2000) [36] (sig 0.985 – 1.039 GeV)
   - CMD-2 (2005) [23] (sig 0.6 – 1.38 GeV)
   - SND (2003) [32] (sig at 0.60 - 0.97 GeV)

   (f) $e^+e^- \to \pi^0\, e^+e^-$ this contains $s$ and $t$ contributions which could be well separated by kinematic cuts:

   - CELLO (1990) [24] $\sqrt{s} = 35$ GeV
   - CLEO (1997) [33] $\sqrt{s} = 10.6$ GeV $-q_1^2 = 1.5 - -9$ GeV$^2$
   - BaBar (2009) [20]
   - Belle (2012) [21]

(g) $e^-e^- \to \pi^0\, e^-e^-$

(h) $e^-\gamma \to \pi^0\, e^-$

(i) $e^+e^- \to \pi^0\, \mu^+\mu^-$

2. Processes with one external (pseudoscalar) hadron: $\eta$

(a) $\eta \to 2\gamma$

(b) $\eta \to \gamma\, e^+e^-$

- SND (2000) [22]
- CB/TAPS (2011) [28]

(c) $\eta \to \gamma\, \mu^+\mu^-$

- Lepton-G (1980)[10]
- NA60 (2009) [15]

(d) $\eta \to e^+e^-\, e^+e^-$:

- KLOE (2011) [11] 362±29 events BR $(2.4 \pm 0.2_{stat+bckg} \pm 0.1_{syst}) \cdot 10^{-5}$.
- CMD-2 (2000)[19] BR UL
- CELSIUS/WASA (2008) [9] BR UL

(e) $\eta \to e^+e^-\, \mu^+\mu^-$

(f) $\eta \to \mu^+\mu^-\, \mu^+\mu^-$

(g) $\eta \to e^+e^-$

(h) $\eta \to \mu^+\mu^-$

(i) $e^+e^- \to \eta\, \gamma$

- SND (1997) [34] 1.020 GeV
- CMD-2 (1999) [35](sig 0.986 – 1.039 GeV) $\eta \to \pi^+\pi^-\pi^0$
- SND (2000) [36] (sig at 0.98513 – 1.03930 GeV)
- CMD-2 (2001) [37] (sig 0.6 – 1.354 GeV) $\eta \to 3\pi^0$
- CMD-2 (2005) [23] (sig 0.6 – 1.38-GeV) $\eta \to \gamma\gamma$
- BaBar (2006) [38] E=10.58 GeV $4.5^{+1.2}_{-1.1}\,{}_{stat} \pm 0.3_{syst}$ fb
- SND (2006) [39] two sets: $\eta \to 3\pi^0$ (0.6-1.36 GeV) and $\eta \to \pi^+\pi^-\pi^0$ (0.755 – 1.056 GeV)
- SND (2007) [40] reanalysis of [39]
- KLOE (2012) [41] $\sqrt{s}$= 1 GeV cross section $856 \pm 8_{stat} \pm 16_{syst}$ pb

P0210 $e^+e^- \to \eta\, e^+e^-$

- CELLO (1990) [24]
- CLEO (1997) [33] $\sqrt{s} = 10.6$ GeV 1.5 – 20 GeV$^2$
- BaBar (2011) [?] $\sqrt{s} = 10.6$ GeV $-40 < q_1^2 < -4$ GeV$^2$
- KLOE (2012) [41] $\sqrt{s}$= 1 GeV cross section t channel, $32.72 \pm 1.27_{stat} \pm 0.70_{syst}$ pb

(j) $e^+e^- \to \eta\,\mu^+\mu^-$

3. Processes with one external (pseudoscalar) hadron: $\eta'$

   (a) $\eta' \to 2\gamma$

   (b) $\eta' \to \gamma\, e^+e^-$

   (c) $\eta' \to \gamma\,\mu^+\mu^-$

   (d) $\eta' \to e^+e^-\, e^+e^-$

   (e) $\eta' \to e^+e^-\,\mu^+\mu^-$

   (f) $\eta' \to \mu^+\mu^-\,\mu^+\mu^-$

   (g) $\eta' \to e^+e^-$

   (h) $\eta' \to \mu^+\mu^-$

   (i) $e^+e^- \to \eta'\,\gamma$

   - BaBar [38] E=10.58

   (j) $e^+e^- \to \eta'$

   - (1988) [12] search for $\eta' \to \eta\pi\pi$ final state

   (k) $e^+e^- \to \eta'\, e^+e^-$

   - CELLO (1990) [24]
   - CLEO (1997) [33] 1.5 to 9, 20, and 30 GeV2
   - L3 (1998) [42]
   - BaBar (2011) [?] $\sqrt{s} = 10.6$ GeV $-40 < q_1^2 < -4$ GeV$^2$

P0311 $e^+e^- \to \eta'\,\mu^+\mu^-$

4. Processes with two external (narrow) hadrons: $\pi^0$ and a vector meson

   (a) $\omega \to \pi^0\,\gamma$

   (b) $\omega \to \pi^0\, e^+e^-$

   (c) $\omega \to \pi^0\,\mu^+\mu^-$

   - Lepton-G (1980) [17]
   - NA60 (2009) [15]
   - NA60 (2011) [16]

   (d) $e^+e^- \to \omega\,\pi^0$

   - ND (1986) [43] $1.0 < E < 1.4$ GeV
   - CMD-2 (1989) [44] $1.05 < E < 1.38$ GeV $\omega \to \pi^+\pi^-\pi^0$
   - DM2 (1990) [45] E 1350-MeV - 2400-MeV
   - SND (2000) [25]: E $< 1.4$ GeV with decay $\omega \to \pi^0\gamma$
   - CMD-2 (2003) [26] 920-MeV - 1380-MeV $\omega \to \pi^0\gamma$
   - CMD-2 (2003) [46] $e^+e^- \to \pi^0\pi^0\gamma$ in c.m. energy range 600-MeV to 970-MeV

- KLOE (2008) [27]
- SND (2012) [47] $1.1 < E < 1.9$
- SND (2013) [48] $1.05 < E < 2.0$ GeV preprint

(e) $\phi \to \pi^0\,\gamma$

(f) $\phi \to \pi^0\,e^+e^-$

- SND (2000) [22]

(g) $\phi \to \pi^0\,\mu^+\mu^-$

(h) $e^+e^- \to \phi\,\pi^0$

5. Processes with two external (narrow) hadrons: $\eta$ and a vector meson

(a) $\omega \to \eta\,\gamma$

(b) $\omega \to \eta\,e^+e^-$

(c) $\omega \to \eta\,\mu^+\mu^-$

(d) $e^+e^- \to \omega\,\eta$

(e) $\phi \to \eta\,\gamma$

(f) $\phi \to \eta\,e^+e^-$

(g) $\phi \to \eta\,\mu^+\mu^-$

(h) $e^+e^- \to \phi\,\eta$

6. Processes with two external (narrow) hadrons: $\eta'$ and a vector meson

(a) $\eta' \to \omega\,\gamma$

(b) $\eta' \to \omega\,e^+e^-$

(c) $e^+e^- \to \omega\,\eta'$

(d) $\phi \to \eta'\,\gamma$

(e) $\phi \to \eta'\,e^+e^-$

(f) $e^+e^- \to \phi\,\eta'$

7. Processes with three external (pseudoscalar) hadrons: $\pi^0\,\pi^+\,\pi^-$

(a) $\pi^\pm\,\gamma \to \pi^0\,\pi^\pm$

(b) $e^+e^- \to \pi^0\,\pi^+\,\pi^-$

8. Processes with three external (pseudoscalar) hadrons: $\eta\,\pi^+\,\pi^-$

(a) $\eta \to \pi^+\pi^-\,\gamma$

- old data...
- WASA-at-COSY (2011) [29] $dN/dE_\gamma$
- KLOE (2012) [30] $dN/dm_{\pi\pi}$

| $q^2$ | range | $\|F_{\pi^0}(q^2,0)\|^2$ | Year/Ref. | comment |
|---|---|---|---|---|
| -0.55 | -0.3 -0.8 | $1.05 \pm 0.24$ | CELLO(1990)[24] | `M(P=3)**2/64/PI*F2` |
| -1.0 | -0.8 -1.2 | $0.56 \pm 0.14$ | CELLO(1990)[24] | `M(P=3)**2/64/PI*F2` |
| -1.45 | -1.2 -1.7 | $0.39 \pm 0.11$ | CELLO(1990)[24] | `M(P=3)**2/64/PI*F2` |
| -2.55 | -1.7 -3.4 | $0.15 \pm 0.05$ | CELLO(1990)[24] | `M(P=3)**2/64/PI*F2` |
| -11.7 | -3.4 -20.0 | $0.027 \pm 0.016$ | CELLO(1990)[24] | `M(P=3)**2/64/PI*F2` |

Table 7: Data on $|F_{\pi^0}(q^2,0)|^2$

(b) $\eta \to \pi^+\pi^-\, e^+e^-$

(c) $\eta \to \pi^+\pi^-\, \mu^+\mu^-$

(d) $\pi^\pm\, \gamma \to \eta\, \pi^\pm$

(e) $e^+e^- \to \eta\, \pi^+\, \pi^-$

9. Processes with three external (pseudoscalar) hadrons: $\eta'\, \pi^+\, \pi^-$

   (a) $\eta' \to \pi^+\pi^-\, \gamma$

   - Data: C- Barrel, L3,...
   - Prospect: BESIII, CLAS

   (b) $\eta' \to \pi^+\pi^-\, e^+e^-$

   (c) $\eta' \to \pi^+\pi^-\, \mu^+\mu^-$

   (d) $\pi^\pm\, \gamma \to \eta'\, \pi^\pm$

   (e) $e^+e^- \to \eta'\, \pi^+\, \pi^-$

Note that the coupling of three neutral pseudoscalars to a photon breaks charge-conjugation invariance. In principle, one could add to the list processes with more external hadrons, e.g., $\eta' \to \pi^+\pi^-\, \pi^+\pi^-$ and $\eta' \to \pi^+\pi^0\, \pi^-\pi^0$ which could proceed via two virtual $\rho$ mesons. For a theoretical discussion of these decays see Ref. [**?**]. The list is restricted to four or less external states, but counting the virtual photon as one state.

# B Decay distributions for $\eta$ and $\eta'$ decays into three pseudoscalar mesons

# C Acknowlegements

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Table 8: The $Q^2$ interval, the weighted average $Q^2$ value for the interval ($\overline{Q^2}$), the $e^+e^- \to e^+e^-\pi^0$ cross section ($\mathrm{d}\sigma/\mathrm{dQ}^2(\overline{Q^2})$), and the product of the $\gamma\gamma^* \to \pi^0$ transition form factor $F(\overline{Q^2})$ and $\overline{Q^2}$. The quoted errors are statistical and systematic for the cross section, and combined for the form factor. In the table we quote only $Q^2$-dependent systematic errors. The $Q^2$-independent systematic error is 3% for the cross section and 2.3% for the form factor.**calculate** $|F|^2$

| $Q^2$ interval (GeV$^2$) | $\overline{Q^2}$ (GeV$^2$) | $\mathrm{d}\sigma/\mathrm{dQ}^2(\overline{Q^2})$ (fb/GeV$^2$) | $\overline{Q^2}\|F(\overline{Q^2})\|$ (MeV) |
|---|---|---|---|
| 4.0-4.5 | 4.24 | 131.4±4.6±5.0 | 150.4±3.9 |
| 4.5-5.0 | 4.74 | 87.7±2.9±3.7 | 149.1±4.1 |
| 5.0-5.5 | 5.24 | 68.4±2.5±2.2 | 157.4±3.9 |
| 5.5-6.0 | 5.74 | 48.3±2.1±1.8 | 156.0±4.5 |
| 6.0-7.0 | 6.47 | 34.8±1.2±1.0 | 163.5±3.6 |
| 7.0-8.0 | 7.47 | 20.01±0.86±0.79 | 160.6±4.7 |
| 8.0-9.0 | 8.48 | 13.60±0.69±0.70 | 167.3±6.0 |
| 9.0-10.0 | 9.48 | 11.11±0.56±0.32 | 185.3±5.5 |
| 10.0-11.0 | 10.48 | 7.73±0.48±0.38 | 186.6±7.6 |
| 11.0-12.0 | 11.49 | 5.86±0.42±0.21 | 191.6±7.8 |
| 12.0-13.5 | 12.71 | 3.35±0.29±0.28 | 175.±11. |
| 13.5-15.0 | 14.22 | 2.82±0.26±0.19 | 198.±12. |
| 15.0-17.0 | 15.95 | 1.99±0.20±0.09 | 208.±12. |
| 17.0-20.0 | 18.40 | 1.27±0.14±0.06 | 220.±13. |
| 20.0-25.0 | 22.28 | 0.73±0.09±0.06 | 245.±18. |
| 25.0-30.0 | 27.31 | 0.18±0.07±0.02 | $181.^{+33.}_{-40.}$ |
| 30.0-40.0 | 34.36 | 0.16±0.04±0.02 | $285.^{+39.}_{-45.}$ |

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