# Homework 1

**Geometrical methods in theoretical physics HT-11**

1. Consider 3-dimensional manifold $M$. In analogy with $\mathbb{R}^3$ define on $M$ the operators of divergence ($div\ \vec{v}$), curl ($rot\ \vec{v}$), and gradient ($grad\ f$) in terms of exterior derivative, $d$, its adjoint $d^\dagger$, etc. You will need to use the metric on $M$ to relate vectors and one-forms, and $*$ to identify two-forms with one-forms. Can you comment on the relations $rot\ grad f = 0$ and $div\ rot\ \vec{v} = 0$? Are they true for a general 3-manifold?

2. Consider the complex projective space $\mathbb{CP}^2$ which is defined as collection of 3 complex coordinates

$$(z_1, z_2, z_3) \in \mathbb{C}^3 - \{0\}$$

together with the identification

$$(z_1, z_2, z_3) \sim (\lambda z_1, \lambda z_2, \lambda z_3)\ , \qquad \lambda \in \mathbb{C}\ , \quad \lambda \neq 0\ .$$

Please, show that $\mathbb{CP}^2$ is a complex manifold by giving explicitly the holomorphic atlas. Explain why $\mathbb{CP}^2$ is a quotient $S^5/U(1)$.

3. Take $\mathbb{R}^3$ and remove the origin. Calculate the de Rham cohomology groups $H^\bullet(M, \mathbb{R})$ for this manifold $M = \mathbb{R}^3 - \{0\}$.

**to be handed in before 5 p.m., January 15, 2012**