Homework 1

Geometrical methods in theoretical physics HT-12

1. Consider the complex projective space $\mathbb{CP}^3$ which is defined as collection of 3 complex coordinates

$$(z_1, z_2, z_3, z_4) \in \mathbb{C}^4 - \{0\}$$

together with the identification

$$(z_1, z_2, z_3, z_4) \sim (\lambda z_1, \lambda z_2, \lambda z_3, \lambda z_4) \; , \qquad \lambda \in \mathbb{C} \; , \quad \lambda \neq 0 \; .$$

Please, show that $\mathbb{CP}^3$ is a complex manifold by giving explicitly the holomorphic atlas. Explain why $\mathbb{CP}^3$ is a quotient $S^7/U(1)$.

2. Take $\mathbb{R}^2$ and remove the origin. Calculate the de Rham cohomology groups $H^\bullet(M, \mathbb{R})$ for this manifold $M = \mathbb{R}^2 - \{0\}$.

to be handed in before 5 p.m., January 15, 2013