

Homework 1

Geometrical methods in theoretical physics HT-14

1. Consider the complex projective space $\mathbb{C}\mathbb{P}^2$ which is defined as collection of 3 complex coordinates

$$(z_1, z_2, z_3) \in \mathbb{C}^3 - \{0\}$$

together with the identification

$$(z_1, z_2, z_3) \sim (\lambda z_1, \lambda z_2, \lambda z_3), \quad \lambda \in \mathbb{C}, \quad \lambda \neq 0.$$

Please, show that $\mathbb{C}\mathbb{P}^2$ is a complex manifold by giving explicitly the holomorphic atlas. Explain why $\mathbb{C}\mathbb{P}^2$ is a quotient $S^5/U(1)$.

2. Take \mathbb{R}^3 and remove the origin. Calculate the de Rham cohomology groups $H^\bullet(M, \mathbb{R})$ for this manifold $M = \mathbb{R}^3 - \{0\}$.

to be handed in before 5 p.m., January 15, 2015