Homework 2

1. Consider the Levi-Civita connection on the manifold M with a Riemannian metric g. Explain the Levi-Civita connection in terms of a connection on principal and associated vector bundles.
(*Hint*: The answer for this question is scattered in the Nakahara's book.)

2. Consider a principal bundle $P(M, G)$ with the connection one-form ω and the corresponding curvature two-form Ω. Show that locally (i.e., over a chart where the bundle can be trivialized) the form $tr(\Omega \wedge \Omega)$ is exact, namely locally

$$tr(\Omega \wedge \Omega) = dK \ .$$

Find this K.

3. Consider the Hopf bundle $S^3 \to S^2$ (see the explanation in the lectures). Describe this principle S^1-bundle in details, give the trivialization and the transition functions. S^3 has a standard metric coming from its embedding into $\mathbb{R}^4$. Using this metric we can define a connection on the Hopf bundle by postulating that the horizontal subspace is the orthogonal complement of the vertical subspace. Explain why this is the case.

Compute the corresponding connection one-form and the curvature two-form on S^3.

to be handed in before 5 p.m., March 15, 2010