

Homework 2

Geometrical methods in theoretical physics HT-12

1. Give two explicit examples of free action of Lie group on smooth manifold.
2. Give two explicit examples of transitive action of Lie group on smooth manifold
3. Consider a Riemannian compact manifold and define Laplace operator

$$\Delta = d^\dagger d + dd^\dagger$$

on differential forms. Are there negative eigenvalues for Δ ? Explain the answer.

the next problem is more complicated. If it is hard for you to solve it you can either ask me for hints or drop it

4. Consider the Hopf bundle $S^3 \rightarrow S^2$ (see the explanation in the lectures). S^3 has a standard metric coming from its embedding into \mathbb{R}^4 . Using this metric we can define a connection on the Hopf bundle by postulating that the horizontal subspace is the orthogonal complement of the vertical subspace. Explain why this is the case.

Compute the corresponding connection one-form and the curvature two-form on S^3 .

to be handed in before 5 p.m., January 15, 2013