Homework 3

1. Consider the connection ∇ on the vector bundle E (see the axiomatic definition from Lectures or Nakahara). Choosing the local basis of sections s^α we can define the connection as

$$\nabla_v s^\alpha = \Gamma^\alpha_{\ \mu\beta}\, v^\mu s^\beta \ , \qquad v \in \Gamma(TM) \ .$$

The basis is changed as $\tilde{s}^\alpha = t^\alpha_{\ \beta}(x) s^\beta$ and the section of the vector bundle is $\sigma = \sigma_\alpha(x) s^\alpha$. Work out the rules $\nabla_v \sigma$ and the transformation rules for Γ. If we define another connection $\tilde{\Gamma}$ on E, then what are the properties of $\Gamma - \tilde{\Gamma}$.

2. Let M be a Kähler manifold. Calculate $c_1(TM^{\mathbb{C}})$.

[*Hint*: look at the page 291 in the Nakahara's book for the expression for the Ricci form and figure out how it is related to $c_1(TM^{\mathbb{C}})$, the first Chern class. Also see the problem 1 in the Homework 2.]

Now assume that $dimM = 2k$ and there exists a globally defined no-where vanishing holomorphic $(k,0)$-form

$$\Omega = e^{f(z)} dz^1 \wedge dz^2 \wedge ... \wedge dz^k \ .$$

Show that $c_1(TM^{\mathbb{C}}) = 0$.

[*Hint*: relate $g = \det(g_{\mu\nu})$ with $||\Omega||$.]

to be handed in before 5 p.m., March 15, 2010