

# Homework 3

## Geometrical methods in theoretical physics HT-11

1. Consider the connection  $\nabla$  on the vector bundle  $E$  over  $M$  (see the axiomatic definition from Lectures or Nakahara). Choosing the local basis of sections  $s^\alpha$  we can define the connection as

$$\nabla_v s^\alpha = \Gamma^\alpha_{\mu\beta} v^\mu s^\beta, \quad v \in \Gamma(TM).$$

The basis is changed as  $\tilde{s}^\alpha = t^\alpha_\beta(x) s^\beta$  and the section of the vector bundle is  $\sigma = \sigma_\alpha(x) s^\alpha$ . Work out the rules  $\nabla_X \sigma$  and the transformation rules for  $\Gamma$ . If we define another connection  $\tilde{\Gamma}$  on  $E$ , then what are the properties of  $\Gamma - \tilde{\Gamma}$ .

2. Using the setting of previous problem, explain and derive the equation for the parallel transport of section  $\sigma$  along a curve  $\gamma : [0, 1] \rightarrow M$ .

**to be handed in before 5 p.m., January 15, 2012**