

Homework 3

Geometrical methods in theoretical physics HT-12

1. Consider the connection D on the vector bundle E over M (see the axiomatic definition from Lectures or Nakahara). Choosing the local basis of sections e^α we can define the connection as

$$D_v e^\alpha = \Gamma^\alpha_{\mu\beta} v^\mu e^\beta, \quad v \in \Gamma(TM).$$

The basis is changed as $\tilde{e}^\alpha = t^\alpha_\beta(x) e^\beta$ and the section of the vector bundle is $s = s_\alpha(x) e^\alpha$. Work out the rules $D_v s$ and the transformation rules for Γ . If we define another connection $\tilde{\Gamma}$ on E , then what are the properties of $\Gamma - \tilde{\Gamma}$.

2. Using above result, derive the explicit expression for the curvature in terms of Γ .

3. Using the setting of first problem, explain and derive the equation for the parallel transport of section s along a curve $\gamma : [0, 1] \rightarrow M$.

to be handed in before 5 p.m., January 15, 2013