

# Homework 4

## Geometrical methods in theoretical physics HT-12

1. Can we have a non-trivial line bundle (or  $S^1$ -bundle) over  $S^3$ ? Explain the answer.
2. Take two sphere  $S^2$  and calculate the following trace over the space of all differential forms  $\Omega^\bullet(S^2)$

$$\text{tr}((-1)^F e^{-\beta\Delta}) ,$$

where  $\Delta$  is Laplace operator on forms.

Do the same for  $S^3$ . Why the answers are different?

[*Hint*: see the lecture notes for the notations]

**to be handed in before 5 p.m., January 15, 2013**