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Intitutionen för Fysik och Astronomi
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Tentamen 20-03-2015, kl. 8-13, Bergsbrunnagatan 15, Sal 1
Fysikens matematiska metoder, KandFy2, KandMa2, Q2 (1FA121)
Hjälpmedel: Physics Handbook, Mathematics Handbook

Comment: regarding the notation for curvilinear coordinates I follow the conventions from Mathematics Handbook.

1. Solve the following partial differential equation

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} - u\,,$$

where $u(x,t)$ satisfies the following boundary conditions

$$u(-1,t) = u(1,t)\,, \qquad \frac{\partial u}{\partial x}(-1,t) = \frac{\partial u}{\partial x}(1,t)\,,$$

and the initial conditions

$$u(x,0) = f(x)\,, \qquad \frac{\partial u}{\partial t}(x,0) = 0\,.$$

2. Consider the heat equation on the disk of radius 2

$$\frac{\partial u}{\partial t} = \nabla^2 u\,.$$

Find a solution $u(\rho,\phi,t)$ which satisfies

$$u(2,\phi,t) = 0\,, \qquad u(\rho,\phi,0) = f(\rho)\cos\phi + g(\rho)\cos 2\phi\,.$$

3. Solve the Laplace equation inside of a sphere of radius 3 with the following boundary condition

$$u(3, \theta, \phi) = \sin\theta \cos^2\theta \sin\phi \,.$$

4. Solve the heat equation with a source

$$\frac{\partial u}{\partial t} = \frac{1}{\pi^2}\frac{\partial^2 u}{\partial x^2} + \cos(2t)\sin(2\pi x)\,,$$

on the interval $[0, 2]$ subject to

$$u(0,t) = 2\,, \qquad u(2,t) = 10\,, \qquad u(x,0) = x^3 + 2\,.$$

5. Find a solution to the Poisson equation

$$\nabla^2 u = f(\rho)\sin(3\pi z) - 2z\,,$$

inside of a cylinder of radius 1 and height 2 with the following boundary conditions

$$u(1,\phi,z) = 0\,, \qquad u(\rho,\phi,0) = 0\,, \qquad u(\rho,\phi,2) = 1 - \rho^2\,.$$