UPPSALA UNIVERSITET

Intitutionen för Fysik och Astronomi

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Fysikens matematiska metoder, F2 (1FA121)
Hjälpmedel: Physics Handbook, Mathematics Handbook

Comment: regarding the notation for curvilinear coordinates I follow the conventions from Mathematics Handbook.

1. Solve the following partial differential equation

$$2\frac{\partial^2 u}{\partial t^2} = 4\frac{\partial^2 u}{\partial x^2} - u\,,$$

where $u(x,t)$ satisfies the boundary conditions

$$\frac{\partial u}{\partial x}(0,t) = 0\,, \qquad u(2,t) = 0\,,$$

and the initial conditions

$$u(x,0) = 0\,, \qquad \frac{\partial u}{\partial t}(x,0) = g(x)\,.$$

2. Solve the Laplace equation inside a cylinder of radius 5 and height 2. Find $u(\rho,\phi,z)$ that satisfies the following boundary conditions

$$u(5,\phi,z) = 0\,, \qquad u(\rho,\phi,0) = f(\rho,\phi)\,, \qquad u(\rho,\phi,2) = g(\rho,\phi)\,.$$

3. Solve the heat equation inside of a sphere of radius 3,

$$\frac{\partial u}{\partial t} = 5\nabla^2 u \,,$$

with the following boundary and initial conditions

$$u(3,\theta,\phi,t) = 0\,, \qquad u(r,\theta,\phi,0) = f(r)\sin^2(\theta)\cos(2\phi) + g(r).$$

4. Solve the wave equation with a source

$$\frac{\partial^2 u}{\partial t^2} = \nabla^2 u + 5e^{-5\pi t}\sin(4\pi x)\sin(3\pi y) + 2y$$

inside the unit rectangle $0 \le x \le 1$, $0 \le y \le 1$. The boundary conditions are

$$\begin{aligned} u(0,y,t) &= 0\,, \\ u(1,y,t) &= 0\,, \\ u(x,0,t) &= 0\,, \\ u(x,1,t) &= x(1-x)\,, \end{aligned}$$

and the initial conditions are

$$u(x,y,0) = f(x,y)\,, \qquad \frac{\partial u}{\partial t}(x,y,0) = 0\,.$$

5. Solve the following partial differential equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + 2\frac{\partial u}{\partial x} + u\,,$$

on the line $-\infty < x < \infty$ with the initial condition

$$u(x,0) = f(x)\,.$$