Department of Physics and Astronomy, Uppsala University

Analytical Mechanics (1FA163), 5 hp

General Information

- Literature: Goldstein, Poole and Safko, *Classical Mechanics, 3rd edition*, Addison Wesley. The chapters that define the course are 8, 9, 10, 12; chapter 11 (classical chaos) is included as an orientation.
- The course consists of 10 lectures and 10 tutorials.
- Examination: At the end of the course there will be a five-hour written exam consisting of 5 problems worth 5 points each.
- Homework: Three hand-in exercises (deadline: Sept 12, 21 and Oct 2 at 5 PM) will give 1 bonus point each to the exam (if solved correctly and on time). The completed homework must be written on paper, and handed in class or in the teachers' mailboxes (3rd floor, outside Hus 8). Homework submitted electronically through email will not be accepted.

Homepage: www.physics.uu.se/research/theoretical-physics/people/henrik-johansson/analytical-mechanics-ht-17

Teachers

- **Lectures:** Henrik Johansson, office 73130, 018-4713243, henrik.johansson@physics.uu.se
- **Tutorials:** Suvendu Giri, office 73134, 018-4713244, suvendu.giri@physics.uu.se

N.B. The plan for the lectures and tutorials may be subject to change.

Plan for the Lectures

1. Course overview. Recapitulation of Newtonian and Lagrangian mechanics. Differentials and Legendre transform. Hamiltonian and Hamilton's equations. Examples: pendulum, free point particle in 1D and 2D.
   Preparatory exercise: Read Chapter 8.1. Remind yourself of concepts such as generalised coordinates and Lagrangian (cf. Chapter 2).
2. Phase portraits. Example: pendulum. Cyclic variables. Conservation laws and Noether's theorem. Examples: conservation of linear and angular momentum.
   Preparatory exercise: Read Chapter 8.2 and 8.5. Derive the Euler-Lagrange equations from variation of the action (cf. Chapter 2).
3. Canonical transformations. Differentials and generating functions. Canonical transformations as symplectomorphisms.
   Preparatory exercise: Read Chapter 9.1 and work through (one of) the examples in Chapter 9.2-9.3. Read Chapter 9.4.
4. Poisson brackets. Relation to conservations laws and conserved quantities.

Preparatory exercise: Read Chapter 9.5-9.6. Consider the different ways of proving that a transformation is canonical (generating function, symplectic approach, Poisson bracket); formulate the pros and cons of the methods!
Post-lecture reading: Note that the developed formalism is useful both in quantum and statistical mechanics. For an example, read Chapter 9.9 on Liouville's theorem.

5. The Hamilton-Jacobi method. Hamilton's principal function and Hamilton's characteristic function. Example: harmonic oscillator. Separation of variables in the Hamilton-Jacobi equation. Example: 2D/3D point particle in a potential.
Preparatory exercise: Read Chapters 10.1-10-3.
6. Hamilton-Jacobi method continued. Action-angle variables.
Preparatory exercise: Read Chapters 10.4-10-5, 10-6-10.8.
7. Action-angle variables continued. Example: 3D Kepler problem. Adiabatic invariants.
Preparatory exercise: Read Chapters 10-6-10.8 and 12.5.
8. Integrable and non-integrable Hamiltonian systems: qualitative behavior. Chaotic systems. Kolmogorov-Arnold-Moser's theorem. Discussion of chaos in the solar system. Perturbation theory.
Preparatory exercise: Read Chapters 11.1-11.4, 12.1.
9. Perturbation theory
Preparatory exercise: Read Chapters 12.1-12.4.
10. Review and questions.

Plan for the Tutorials

The problems discussed at each tutorial can be found on the course homepage (L) or in Goldstein (G). You are expected to have solved (attempted to solve) these problems before class, so that there can be a useful discussion (You may be asked to demonstrate your solutions on the blackboard).

1. Legendre transformations between Lagrangian and Hamiltonian. Equivalent Lagrangians and their corresponding Hamiltonians. (L1:1-4)
2. Legendre transformations between Lagrangian and Hamiltonian. Cyclic variables. Phase portraits for 1D systems. (L2:1-4, L3:1)
3. Canonical transformations (L3:2-3,L4:1).
4. Canonical transformations. Poisson brackets (L4:2-4,L5:1).
5. Poisson brackets. Conserved quantities (L5:2-3,L6:1).
6. Hamilton-Jacobi method. Hamilton's principle and characteristic functions. Separable problems. (L6:2-4,L7:1-2)
7. Hamilton-Jacobi method. Action-angle variables. (L7:3-4,L8:1-2)
8. Adiabatic invariants. Perturbation theory. (L9:1-4,G:12:2)
9. Old exam
10. Q&A